(1) $f(x) = 3x^2 + 7x$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{3(x+h)^2 + 7(x+h) - (3x^2 + 7x)}{h}$$

$$\lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) + 7(x+h) - (3x^2 + 7x)}{h}$$

$$\lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 + 7x + 7h - 3x^2 - 7x}{h}$$

$$\lim_{h \to 0} \frac{6hx + 3h^2 + 7h}{h}$$

$$\lim_{h \to 0} \frac{h(6x + 3h + 7)}{h} \rightarrow h = 0 \quad 6x + 3h + 7$$

$$6x + 3(0) + 7 = \boxed{6x + 7}$$

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(2) $f(x) = x^3 - 5$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^3 - 5 - (x^3 - 5)}{h}$$

$$\lim_{h \to 0} \frac{x^3 + 3h^2x + 3hx^2 + h^3 - 5 - x^3 + 5}{h}$$

$$\lim_{h \to 0} \frac{3h^2x + 3hx^2 + h^3}{h}$$

$$\lim_{h \to 0} \frac{h(3hx + 3x^2 + h^2}{h}$$

$h = 0 \quad 3hx + 3x^2 + h^2$

$3(0)(x) + 3x^2 + 0^2$

$0 + 3x^2 + 0$

$\boxed{3x^2}$

(3) $f'(x) = 10x + 12$

(4) Chain Rule:

P T A

$5\tan^4(3x^2)\ \sec^2(3x^2)\ (6x)$

$30x\ \tan^4(3x^2)\sec^2(3x^2)$

(5) $y' = 24x^2 + 5$

(6) $f(x) = \underbrace{(2x+3)}_{a}\underbrace{\left(\frac{3x^3}{\sin x}\right)}_{b}$

Product Rule

$a\ b' + b\ a'$

$(2x+3)\left(\frac{ba' - ab'}{b^2}\right) + \left(\frac{3x^3}{\sin x}\right)(2)$

Quotient Rule

~~Inside~~ ( )

$(2x+3)\left(\frac{(\sin x)(9x^2) - (3x^3)(\cos x)}{(\sin x)^2}\right) + \left(\frac{3x^3}{\sin x}\right)(2)$

(7) $f(x) = (4x^2+5)^2$

Option 1

Chain Rule

P T A

$2(4x^2+5)^1 (\quad)(8x)$

$16x(4x^2+5)$

$64x^3+80x$

Option 2 → Rewrite Original

$(4x^2+5)(4x^2+5)$

$16x^4+20x^2+20x^2+25$

Original $f(x) = 16x^4+40x^2+25$

$f'(x) = 64x^3+80x$

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(8) Slope of tangent line = derivative

$y' = 8x^3+5$

plug in 4

$8(4)^3+5 = 517$

(9) Same as question 8

$y' = 5x^4+21x^2$

plug in -1

$5(-1)^4+21(-1)^2$

26

$y' = 9x^2+2$

(10) $y-y_1 = m(x-x_1)$

$y-\underline{-28} = \underline{38}(x-\underline{-2})$

$y+28 = 38(x+2)$

$y+28 = 38(x+2)$

(11) $y-y_1 = m(x-x_1)$

$y' = 4x^3+5$

$y-\underline{96} = \underline{113}(x-\underline{3})$

$y-96 = 113(x-3)$